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Calculation of Chirality Violating Proton Structure Function $h_1(x)$ in QCD

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Abstract

The twist-two chirality violating proton structure function $h_1(x)$ measurable in the polarized Drell-Yan process is calculated by means of QCD sum rules at intermediate x, $0.3 < x < 0.7$ and $Q^2 \approx 5 - 10 GeV^2$.

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1 Introduction

As is well known, all structure functions of the lowest twist-two, $F_1(x), F_2(x), g_1(x)$ which are measured in the deep-inelastic lepton-nucleon scattering, conserve chirality. Ralston and Soper [1] first demonstrated that besides these structure functions, there exists the twist-two chirality violating nucleon structure function $h_1(x)$. This structure function does not manifest itself in the deep inelastic lepton-hadron scattering, but can be measured in the Drell-Yan process with both beam and target transversely polarized. The reason of this circumstance is the following. The cross section of the deep inelastic electron(muon)-hadron scattering is proportional to the imaginary part of the forward virtual photon-hadron scattering amplitude. At high photon virtuality the quark Compton amplitude dominates, where the photon is absorbed and emitted by the same quark (Fig.1a) and the conservation of chirality is evident. The cross section of the Drell-Yan process can be represented as an imaginary part of the Fig. 1b diagram. Here virtual photons interact with different quarks and it is possible, as is shown in Fig.1b, that their chiralities are also different. It is clear from Fig.1b, that chirality violating amplitude in Drell-Yan processes is not suppressed at high Q^2 in comparison with chirality conserving ones and, consequently, corresponds to twist two. However, this amplitude corresponding to target spin flip, has no parton interpretation in the standard helicity basis. Nevertheless, $h_1(x)$ can be interpreted as a difference of quark densities with eigenvalues $+1/2$ and $-1/2$ of the transverse Pauli-Lubanski operator $\hat{S}_\perp\gamma_5$ in the transversely polarized proton as it is explained in details by Jaffe and Ji in [2].

Until now, there is no experimental data on the chirality violating nucleon structure function $h_1(x)$. Besides [1, 2] the theoretical study of this structure function (under the name $\Delta_1 q(x)$) have been performed also by Artru and Mekhfi [3]. The role of $h_1(x)$ in factorization of a general hard process with polarized particles was investigated by Collins [4]. The first attempt to calculate $h_1(x)$ was carried out by Jaffe and Ji [2] by means of the bag model.

In this paper we calculate the proton structure function $h_1^p(x)$ by means of the QCD sum rule approach. The idea of the method is the following [5, 6]. Consider the four-point vacuum correlator corresponding to the amplitude of forward scattering of the current $\eta(x)$ with proton quantum numbers on external currents $j_1(x), j_2(x)$. (In the case of the structure function $F_2(x)$ considered earlier [5, 6] $j_1(x), j_2(x)$ were electromagnetic or weak currents). Let the momenta, corresponding to the currents $\eta(x)$ and $j(x)$ be p and q. As was shown in [5, 6], the imaginary part of the forward scattering amplitude is determined by small distances in t-channel if p^2 and q^2 are negative and large enough, $\mid p^2 \mid, \mid q^2 \mid \gg R^{-2}$, (R is the confinement radius) and the Bjorken scaling variable $x = Q^2/2\nu, \quad Q^2 = -q^2, \quad \nu = p \cdot q$ is not close to the boundary values $x = 0$ and $x = 1$. Therefore, to calculate the amplitude of interest we may use the operator product expansion (OPE) method accounting the vacuum expectation values (v.e.v.) of various operators. If, in addition, we suppose $\mid p^2 \mid \ll \mid q^2 \mid$ and restrict ourselves to the first term in expansion over p^2/q^2, only twist-two structures will be retained. In order to obtain the desired proton structure function we use the standard procedure of the QCD sum rule approach. The amplitude calculated in QCD is equated to the contribution of physical states, the proton and the excited states with proton quantum numbers, by means of the dispersion relation in p^2. The proton contribution we are interested in, is then separated by applying the Borel transformation in p^2 which suppresses the contribution of excited states.

It is important to check that the results satisfy necessary conditions of QCD sum rule calculations: 1) the OPE is converging i.e., the the highest accounted term in OPE is well

less than the first term(s) of OPE and 2) in the phenomenological part of the sum rule the contribution of excited states does not surpass the one of the proton. These requirements determine the domain of x, where the results are valid. The value of Q^2 must be much larger than the proton mass m squared, $Q^2 \gg m^2$, but cannot be very large, since the leading logarithmic corrections in QCD are not accounted in our approach. So, we expect that our results are valid at $Q^2 \sim 5 - 10GeV^2$.

It should be emphasized that calculation of various proton structure functions by means of QCD sum rules contains no experimental inputs. Moreover, there are no new parameters at all. The values of vacuum condensates determining the OPE are fixed from QCD sum rules for various two-point correlators, whereas the parameters of the proton channel, namely the transition constant of proton into the quark current and the value of the continuum threshold are taken from the analysis of the sum rules for the proton mass [7]. The price for model independence is a limited domain of x where the calculated structure function is reliable. Thus, for the valence $u(x)$ distribution calculated in [6] the region of validity $0.2 < x < 0.7$ was established. Calculated by the same method, the u quark contribution to the proton spin structure function $g_1^p(x)$ [8] has an even narrower interval of validity $0.5 < x < 0.7$. The criteria determining these intervals as well as the accuracy of the calculation are well defined from comparing nonleading and leading terms of OPE, as explained above. A comparison with experiment shows that one can rely on this method of calculation within the domain of validity. A very nontrivial evidence of that is the accordance of the calculated value of $g_1^p(x)$ with recent experimental data (see detailed discussion below).

In carrying out this program for calculation of $h_1(x)$ some specifics arise. In refs.[1, 2] the structure function $h_1(x)$ was represented in terms of the matrix elements of the operators on the light-cone. In our calculation we start from the forward scattering amplitude on the proton. In Sec. 2 the forward scattering amplitude corresponding to the light cone definition of $h_1(x)$ is constructed. The four-point current correlator calculated in QCD contains many spin-tensor structures and it is necessary to find such structure, which, if saturated by the proton state, gives the desired function $h_1(x)$. This is done in Sec.3.

In sec.4 we calculate the OPE of the four-point current correlator. Here the most important difference between the function $h_1(x)$ and other twist-two structure functions appears: since $h_1(x)$ violates chirality, the OPE in the chosen spin-tensor structure starts from the operator of dimension 3 - the quark condensate, unlike the case of other twist-two structure functions, where it starts from the zero dimension unit operator. In addition to the main term of OPE we calculate the next term proportional to the mixed quark-gluon condensate of dimension 5.

In Sec.5 we obtain the sum rule, perform numerical analysis and give the results of the $h_1(x)$ determination. Our conclusions are presented in Sec.6.

2 The connection of light-cone and forward scattering amplitude definitions of the structure function $h_1(x)$

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The structure function $h_1(x)$ was defined in ref.[2] by Fourier transformation of the operator product on the light cone between proton states

$$i\int\frac{d\lambda}{2\pi}e^{i\lambda x}<p,s\mid\bar{\psi}(0)\sigma_{\mu\nu}\gamma_5\psi(\lambda n)\mid p,s>=$$
$$2[h_1(x,q^2)(s_{\perp\mu}p_\nu-s_{\perp\nu}p_\mu)+h_L(x,q^2)m^2(p_\mu n_\nu-$$
$$p_\nu n_\mu)(s\cdot n)+h_3(x,q^2)m^2(s_{\perp\mu}n_\nu-s_{\perp\nu}n_\mu)] \quad (1)$$

Here n is a light cone vector of dimension of $(mass)^{-1}$, $n^2=0$, $n^+=0$, $p\cdot n=1$, p and s are the proton momentum and spin vectors: $p^2=m^2$, $s^2=-1$, $p\cdot s=0$, and $s=(s\cdot n)p+(s\cdot p)n+s_\perp$. We choose in this section the proton and virtual photon c.m.system and put $q=\{q_0,0,0,E\}$, $p=\{E,0,0,-E\}$, $\quad n=\{1/E,0,0,1/E\}/2$, where E is the virtual photon momentum in this system and the terms of order m^2/E^2 are neglected.

Let us demonstrate now that $h_1(x)$ can also be defined as a matrix element between proton states for forward scattering of the axial current with its transition into scalar current $pA\to pS$ plus crossing matrix element [1]

$$T_\mu(p,q,s,s)=i\int d^4xe^{iqx}<p,s\mid(1/2)T\{j_{\mu5}(x),j(0)+j(x),j_{\mu5}(0)\}\mid p,s> \quad (2)$$

where $j_{\mu5}(x)$ and $j(x)$ are axial and scalar currents. Supposing approximate conservation of axial current, $T_\mu(p,q,s,s)$ can be represented as

$$T_\mu(p,q,s,s)=(s_\mu-\frac{q\cdot s}{q^2}q_\mu)\tilde{h}_1(x,q^2)+ \quad (3)$$
$$+(p_\mu-\frac{\nu q_\mu}{q^2})(q\cdot s)l_1(x,q^2)+\epsilon_{\mu\nu\lambda\sigma}p_\nu q_\lambda s_\sigma(q\cdot s)l_2(x,q^2)$$

(only spin-dependent terms are retained). We will prove that

$$h_1(x,q^2)=-\frac{1}{\pi}Im\tilde{h}_1(x,q^2) \quad (4)$$

up to higher twist terms.

Until now we did not specify the flavors of axial and scalar currents. Strictly speaking, due to the anomaly, the representation (3) is correct only for flavor octet axial current. For the singlet axial current additional terms appear in (3). But, as we shall see, in proton the contribution of u-quarks to $h_1(x)$ is much larger than of d-quarks. Therefore, there is no matter whether we consider $\bar{u}u$ current, as we shall do, or $\bar{u}u-\bar{d}d$ current, where anomaly does not contribute and (3) is correct up to the terms of order m^2/Q^2. In any case, the anomaly contribution is suppressed by α_s, what is beyond the accuracy of our results. For these reasons, we use eq.(1) for the case of one flavor.

As is well known, at high Q^2 the imaginary part of the forward scattering amplitude like (2) is dominated by the light cone region in space-time. Using the expression for quark propagator near light cone

$$<0\mid T\{\psi_\alpha(x),\bar{\psi}_\beta(0)\}\mid 0>=-\frac{i}{4\pi^2}(\gamma_\mu)_{\alpha\beta}\frac{\delta}{\delta x_\mu}\frac{1}{x^2-i\varepsilon} \quad (5)$$

and substituting it into (2) we obtain

$$ImT_\mu=-\frac{1}{4\pi}\int d^4xe^{iqx}<p,s\mid\bar{\psi}(x)(\hat{x}\gamma_\mu-\gamma_\mu\hat{x})\gamma_5\psi(0)\mid p,s>\delta'(x^2) \quad (6)$$

[1] We are indebted to R.Jaffe for the idea of this definition of $h_1(x)$.

Using the light cone variables

$$t - z = \tau \quad (1/2)(t + z) = \lambda \tag{7}$$

we can rewrite (6) as

$$ImT_\mu = -\frac{i}{2}\int d\lambda \int d\tau \int dx^2 e^{iE(\tau - 2\xi\lambda)}\theta(2\tau\lambda - x^2)\delta'(x^2)\times \tag{8}$$
$$\times < p,s \mid \bar{\psi}(x)\sigma_{\mu\nu}x_\nu\gamma_5\psi(0) \mid p,s >$$

where the notation ξ was temporarily accepted for the Bjorken scaling variable and we used the relation

$$qx \approx E(t-z) + \frac{q^2}{2E}t \approx E\tau - 2\xi E\lambda \tag{9}$$

Integrating over x^2 and τ in (8) and putting on the light cone

$$x_\nu = 2E\lambda n_\nu \tag{10}$$

we get

$$ImT_\mu = -\frac{i}{4}\int\limits_{-\infty}^{\infty} d\lambda' e^{i\lambda'\xi} < p,s \mid \bar{\psi}(0)\sigma_{\mu\nu}n_\nu\gamma_5\psi(n\lambda') \mid p,s >, \tag{11}$$

where $\lambda' = 2E\lambda$. Equation (11) can be compared with (1), multiplied by n_ν. The comparison of terms, proportional to s_μ, gives eq.(4). It should be mentioned that instead of axial and scalar currents one can also use a combination of vector and pseudoscalar currents $j_\mu(x)j_5(0) - j_5(x)j_\mu(0)$.

3 Selection of spin-tensor structures

Consider the four-point correlator

$$\Pi_\mu(p,q) = -i\int d^4x d^4y d^4z e^{iqx+ip(y-z)}\times \tag{12}$$
$$\times < 0 \mid T\{\eta(y), 1/2(j_{\mu 5}(x), j(0)+j(x), j_{\mu 5}(0)), \bar{\eta}(z)\} \mid 0 >$$

where η is the three-quark current with the proton quantum numbers [7]

$$\eta = \varepsilon^{abc}(u^a C\gamma_\lambda u^b)\gamma_5\gamma_\lambda d^c \tag{13}$$

$a, b, c = 1, 2, 3$ are colour indices. We are interested in the proton contribution to $\Pi_\mu(p,q)$, which is given by

$$\Pi_\mu^{(p)}(p,q) = \lambda_N^2 \frac{1}{(p^2-m^2)^2}\sum_{r,r'} v^r(p)T_\mu^{(p)}(p,q,r,r')\bar{v}^{r'}(p) \tag{14}$$

where $v^r(p)$ is the proton spinor with momentum and polarization r, λ_N is the transition constant of proton into quark current

$$< 0 \mid \eta \mid p, r >= \lambda_N v^r(p)$$

and $T_\mu^{(p)}(p,q,r,r')$ is nondiagonal in the proton spin matrix element (2). It is convenient to represent $T_\mu^{(p)}(p,q,r,r')$ in the form

$$T_\mu^{(p)}(p,q,r,r') = \bar{v}^r(p)\tilde{T}_\mu^{(p)}(p,q)v^{r'}(p), \tag{15}$$

Here $\tilde{T}_\mu^{(p)}(p,q)$ is the matrix element between proton states before multiplication on proton spinors, i.e. where $p^2 = m^2$, but $\hat{p} \neq m$.

Since the axial and scalar currents enter symmetrically (2) and (12), $\Pi_\mu(p,q)$ as well as $\tilde{T}_\mu^{(p)}(p,q)$ satisfy the condition

$$F(q,p) = C^{-1}F^T(-q,-p)C, \quad F = \Pi_\mu, \quad \tilde{T}_\mu^{(p)} \tag{16}$$

where C is the charge conjugation matrix. Accounting for (16) the general structure of $\tilde{T}_\mu^{(p)}(p,q)$ is

$$\begin{aligned}\tilde{T}_\mu^{(p)}(p,q) = [K(\hat{q}\gamma_\mu - \gamma_\mu\hat{q}) + E(\hat{p}\hat{q} - \hat{q}\hat{p})p_\mu + Cp_\mu\hat{q} + C'q_\mu\hat{p} \\ +D(\hat{p}\hat{q} - \hat{q}\hat{p})q_\mu + L(\hat{p}\gamma_\mu - \gamma_\mu\hat{p}) + M\gamma_\mu + R\hat{q}q_\mu + S\hat{p}p_\mu]\gamma_5\end{aligned} \tag{17}$$

where $K, E, C, C', D, L, M, R, S$ are functions of $\nu = p\cdot q, q^2$ and $p^2 = m^2$. $\Pi_\mu(p,q)$ has the same structure, in this case p^2 being not equal to m^2.

The matrix element of $\tilde{T}_\mu^{(p)}(p,q)$ over the polarized proton state with spin s_μ is given by

$$(1/2)Tr\{\tilde{T}_\mu^{(p)}(p,q)(1-\gamma_5\hat{s})(\hat{p}+m)\} = -4s_\mu[\nu K + Lm^2 + (1/2)Mm] + ... \tag{18}$$

where only the terms proportional to s_μ are written explicitly. In accord with (3) these terms are proportional to $h_1(x)$. Substitute (15) into (14) and use the relation

$$\sum_r v_\alpha^r(p)\bar{v}_\beta^r(p) = (\hat{p}+m)_{\alpha\beta}$$

We have

$$\begin{aligned}\Pi_\mu^{(p)} = \frac{\lambda_N^2}{(p^2-m^2)^2}(\hat{p}+m)\tilde{T}_\mu^{(p)}(p,q)(\hat{p}+m) = \frac{\lambda_N^2}{(p^2-m^2)^2}\times \\ \{-\frac{4m}{\nu}[(\nu K + Lm^2 + (1/2)Mm) + m^2q^2(D+(1/2)R)]p_\mu\hat{q} \\ + 4m^3(D+(1/2)R)\hat{q}q_\mu\}\gamma_5 + ...\end{aligned} \tag{19}$$

where the terms at the structures $p_\mu\hat{q}\gamma_5$ and $q_\mu\hat{q}\gamma_5$ in the decomposition (17) are separated. When deriving (19) we used the relation $\Pi_\mu q_\mu = 0$ following from conservation of axial current. As is seen from (18), (19) the combination of the structure functions $\nu K + Lm^2 + Mm/2$, which is proportional to $h_1(x)$ can be found by studying $\Pi_\mu^{(p)}(p,q)$ as a coefficient function at the structure $p_\mu\hat{q}\gamma_5$ added by the coefficient function at the structure $q_\mu\hat{q}\gamma_5$, multiplied by $q^2/\nu = -2x$. In fact, the latter, as can be easily seen, and will be proved by direct calculation in Sec.4 corresponds to twist 4 and can be neglected.

Therefore, the recipe of the calculation of $h_1(x)$ is the following. Calculate $Im\Pi_\mu(p,q)$ defined by eq.(12) in QCD at $p^2 < 0$ using OPE in $1/p^2$ and retaining the leading

terms in $1/q^2$, corresponding to twist 2 amplitude. Separate the coefficient function at the structure $p_\mu \hat{p}\gamma_5$. On the other side, using the dispersion relation in p^2 represent the same function through the contribution of the physical states - the proton and excited states, the latter approximated by continuum. Equate two representations and apply the Borel transformation to both sides of the sum rule in order to enhance the proton contribution and improve the convergence of OPE. This (up to some details) gives the desired sum rule for determination of the proton structure function $h_1(x)$.

4 QCD calculation of four-point correlator

We calculate the imaginary part in s-channel of the four-point correlator $\Pi_\mu(p,q)$ (12). Since $\Pi_\mu(p,q)$ violates chirality, the OPE starts from the term proportional to quark condensate $< 0 \mid \bar{q}q \mid 0 >$. The corresponding diagrams are given in Fig.2.

Consider first the case, when the axial and scalar currents are u-quark currents (Fig.2a). The calculation of the diagram Fig.2a gives

$$Im\Pi^u_\mu(p,q) = -\frac{1}{4\pi\nu} < 0 \mid \bar{u}u \mid 0 > [(-4 + 9x + \\ + 2xln\frac{-p^2x}{2\nu})p_\mu\hat{q} + q_\mu\hat{q}]\gamma_5 \tag{20}$$

(only the structures $p_\mu\hat{q}\gamma_5$ and $q_\mu\hat{q}$ are retained). After the Borel transformation in p^2 the contribution of the structure $q_\mu\hat{q}\gamma_5$ vanishes and we get

$$B_{M^2}Im\Pi^u_\mu(p,q) = \frac{1}{2\pi\nu} < 0 \mid \bar{u}u \mid 0 > M^2 x p_\mu\hat{q}\gamma_5 \tag{21}$$

where M^2 is the Borel parameter.

In the case, when the scattering proceeds on d-quark, it is clear that the corresponding amplitude (Fig.2b) in the lowest twist depends only on ν and q^2 and is p^2 independent. (Direct calculation gives the value $-(1/12\pi) < 0 \mid \bar{d}d \mid 0 > /\nu x$ for the term proportional to $p_\mu\hat{q}\gamma_5$). Thus, it vanishes after Borel transformation. The nonvanishing contribution in the case of scattering comes from diagrams with additional gluon exchange in the diagram of Fig.1b. They are of the order (α_s/π) times the r.h.s. of eq.(21) and we will disregard them, since they are of the same order as perturbative corrections to eq.(21), which are neglected.

The next term in the OPE for $\Pi_\mu(p,q)$ is proportional to the mixed quark-gluon condensate of dimension 5:

$$-g < 0 \mid \bar{q}\sigma_{\mu\nu}G^n_{\mu\nu}\frac{\lambda^n}{2}q \mid 0 > \equiv m_0^2 < 0 \mid \bar{q}q \mid 0 >, \tag{22}$$

Consider again first the case, when the scattering proceeds on u-quarks. The calculations will be performed in the fixed point gauge

$$x_\mu A^n_\mu(x) = 0 \tag{23}$$

and we choose as a fixed point the left-side lower vertex point in Figs.1,2. The first source of appearance of quark-gluon condensate is the expansion of quark condensate in the diagram of Fig.2a in powers of x. We write now

$$< 0 \mid T\{u^a_\alpha(x), \quad \bar{u}^b_\beta(0)\} \mid 0 >= -(1/12)\delta^{ab}\delta_{\alpha\beta} < 0 \mid \bar{u}(0)u(0) \mid 0 > +$$
$$+ \frac{1}{3\cdot 2^7}\delta^{ab}\delta_{\alpha\beta} g < 0 \mid \bar{u}(0)\sigma_{\mu\nu}G^n_{\mu\nu}\lambda^n u(0) \mid 0 > x^2 \qquad (24)$$

The first term in the r.h.s. of eq.(24) gives the already accounted quark condensate contribution. After substituting the second term in (24) into the diagram of Fig.2a we obtain for the coefficient function at the structure $p_\mu \hat{q}\gamma_5$

$$Im\Pi_\mu = \frac{1}{4\pi\nu}\frac{m_0^2}{p^2} < 0 \mid \bar{u}u \mid 0 > \frac{x}{2} p_\mu \hat{q}\gamma_5 \qquad (25)$$

The structure $q_\mu \hat{q}\gamma_5$ is absent in twist 2 terms. In other way the quark-gluon condensate emerges from the diagrams of Fig.3. To calculate these diagrams we used the quark propagator in the constant gluonic field [6]:

$$S(x,z) = \frac{i}{(2\pi)^4}\int d^4k e^{-ik(x-z)}\{\frac{\hat{k}}{k^2} - \frac{1}{4}g\lambda^n G^n_{\alpha\beta}\varepsilon_{\alpha\beta\sigma\rho}\gamma_5\gamma_\rho\frac{k_\sigma}{k^4} +$$
$$+ \frac{1}{4}g\lambda^n G^n_{\alpha\beta} z_\beta(\gamma_\alpha k^2 - 2k_\alpha\hat{k})\frac{1}{k^4}\} \qquad (26)$$

The results of the calculations of the Fig.3 diagrams are the following. The diagrams with gluon emission from both horizontal lines of u and d-quarks (Figs.3a,b) give a vanishing contribution to the coefficients at the structures we are interested in. The sum of the diagrams Figs.3c,d with gluon emission from vertical u-quark lines, when the second term in the r.h.s. of (26) is accounted, gives

$$Im\Pi_\mu = -\frac{1}{4\pi\nu}\frac{m_0^2}{p^2} < 0 \mid \bar{u}u \mid 0 > \frac{1}{6}(\frac{1-x}{x}) p_\mu \hat{q}\gamma_5 \qquad (27)$$

The most complicated is the calculation of the third term in the r.h.s. of the (26) contribution. In the chosen fixed point this term contributes only to the Fig.3d diagram and the calculation gives:

$$Im\Pi_\mu = -\frac{1}{4\pi\nu}\frac{m_0^2}{p^2} < 0 \mid \bar{u}u \mid 0 > \frac{1}{12} p_\mu \hat{q}\gamma_5 \qquad (28)$$

For all Fig.3 diagrams the coefficient function at the structure $\hat{q}q_\mu\gamma_5$ vanishes in twist 2 terms. The total contribution of dimension 5 mixed quark-gluon condensate to the four-point correlator, when the scattering proceeds on u-quarks, is obtained by summing (25),(27),(28)

$$Im\Pi^u_\mu = -\frac{1}{4\pi\nu}\frac{m_0^2}{p^2} < 0 \mid \bar{u}u \mid 0 > \frac{1}{6}(\frac{1}{x} - \frac{1}{2} - 3x) p_\mu \hat{q}\gamma_5 \qquad (29)$$

After borelization we have

$$B_{M^2} Im\Pi^u_\mu(p,q) = \frac{1}{4\pi\nu} m_0^2 < 0 \mid \bar{u}u \mid 0 > \frac{1}{6}(\frac{1}{x} - \frac{1}{2} - 3x) p_\mu \hat{q}\gamma_5 \qquad (30)$$

For the case of scattering on d-quark the contribution of quark-gluon condensate vanishes for the same reason as the quark condensate contribution. Since in the scattering on d-quark the first two terms in OPE vanish and the charge square of d-quark is 4 times smaller than

u-quark, we can safely disregard the d-quark contribution to the proton structure function $h_1(x)$.

5 The sum rule. Determination of $h_1(x)$.

In accordance with the results of Sec.3 consider the coefficient function $H(p^2,x)$ at the structure $p_\mu \hat{q}\gamma_5$ in the four-point correlator (12) as a function of p^2. Using the double dispersion relation represent $H(p^2,x)$ in terms of the contribution of physical states - the proton and excited states

$$H(p^2,x) = -\frac{\pi m}{\nu}\frac{\lambda_N^2}{(p^2-m^2)^2}h_1(x) + \frac{\pi m}{\nu}\frac{A(x)}{p^2-m^2} + \int\limits_{W^2}^{\infty}\frac{\rho(p_1^2,x)}{(p_1^2-p^2)^2}dp_1^2 \tag{31}$$

The factor $-\pi$ in the first term in (31) comes from (4) and the factor m/ν emerges from comparison of (18) and (19). The last term in (31) corresponds to the contribution of excited states approximated by continuum. In our case this contribution is determined by the diagram of Fig.1a and is given by eq.(20), W^2 is the continuum threshold. The second term in (31) is a nondiagonal term where the current $\bar{\eta}$ produces a proton from the vacuum, which, after interaction with $j_{\mu 5}$ and j goes into some excited state N^* absorbed finally by the current η (for details see [6]). Apply the Borel transformation to (31) and equate it to the result of QCD calculations (21),(30). We obtain the sum rule for u-quark contribution to the proton structure function $h_1^p(x)$

$$\frac{h_1^u(x)}{M^2} + B(x) = 2a\frac{e^{m^2/M^2}}{\tilde{\lambda}_N^2 m}[2M^2 x E_1(\frac{W^2}{M^2})L^{-4/9} \tag{32}$$

$$+\frac{m_0^2}{6}(\frac{1}{x}-\frac{1}{2}-3x)L^{-8/9}],$$

where $B = A/\lambda_N^2$,

$$a = -(2\pi)^2 < 0 \mid \bar{q}q \mid 0 >= 0.55GeV^2$$
$$\tilde{\lambda}_N^2 = 32\pi^4\lambda_N^2 = 2.1GeV^6, \quad W^2 = 2.3GeV^2 \tag{33}$$
$$E_1(z) = 1 - e^{-z}(1+z)$$

and powers the of factor $L = ln(M/\Lambda)/ln(\mu/\Lambda)$ take into account anomalous dimensions of currents in the correlator and operators in the OPE. (see [7, 9] for details). Note that the anomalous dimension of the scalar current cancels with the corresponding anomalous dimension of the quark condensate operator in the first term of (33), whereas the anomalous dimension of the quark-gluon condensate operator is small and we neglect it. We assume $\Lambda = 150MeV$ and $\mu = 0.5GeV$. The numerical values in (33) correspond to the best description of nucleon mass and magnetic moments in QCD sum rule approach [9, 10]. In order to get rid of unknown contribution of $B(x)$ let us differentiate (32) over $1/M^2$. We get

$$h_1^u(x) = 2\frac{a}{m\tilde{\lambda}_N^2}e^{m^2/M^2}\{2M^2x[(m^2-M^2)E_1(\frac{W^2}{M^2}) + \frac{W^4}{M^2}e^{-W^2/M^2}]L^{-4/9} +$$

$$+\frac{1}{6}m_0^2m^2(\frac{1}{x}-\frac{1}{2}-3x)L^{-8/9}\} \tag{34}$$

As is seen from (32),(34), these expressions are not correct at $x \to 0$, the OPE breaks down at small x, as is expected from general grounds [6]. In order to find the values of x, where we can believe our results, compare the magnitude of the second term of OPE with the first one. We find that at $x \geq 0.4$ the second term of OPE comprises less than 20-25% of the total in eq.(34) and less than 15-20% in eq.(32). Therefore we may have a confidence in our results at $x > 0.3$. At x close to 1 our results are not reliable because it is a resonance region (see ref. [6] for more detailed explanation). In Fig.4 we plot the Borel parameter M^2 dependence of $h_1^u(x)$ determined according to eq.(34). As is seen from the Figure, the dependence is rather weak and this necessary condition for the QCD sum rule validity is fulfilled. Our basic result - the u-quark contribution to proton structure function $h_1^u(x)$ determined according to (34), is plotted as a solid curve in Fig.5 (at $M^2 = 1GeV^2$).

It must be mentioned that differentiation over $1/M^2$, which was used in order to get (34) from (32), spoils the accuracy of the sum rule, since the role of continuum and of unaccounted higher order terms of OPE increases. Particullarly, the continuum contribution, which comprises 30% in eq.32 is about 70% or even more, especially at large x in eq.(34). (these values refer to $M^2 = 1GeV^2$). For this reason and also because we accounted only two terms of OPE, the accuracy of our results is not high. We estimate it by $\sim 30\%$ at $x \approx 0.5$ and by 50% at the border of interval where our approach is working, $x = 0.3$ and $x = 0.7$. At $x > 0.7$ the curve of $h_1^u(x)$ given in Fig.4 is not correct, since it violates the inequality $\mid h_1^u(x) \mid < u(x)$ proved in ref. [2].

As was explained above, the contribution of d-quarks is small and the proton structure function

$$h_1^p(x) \approx (4/9)h_1^u(x). \tag{35}$$

In Fig.5 we also plotted $h_1^u(x)$ found by Jaffe and Ji in the bag model [2] (dashed curve). The bag model curve is 2-3 times higher compared with the results of our calculations and strongly depends on x at intermediate x, while our $h_1^u(x)$ is flat (due to errors we cannot insist on negative curvature of $h_1^u(x)$, probably it is an artifact of our approximations). It should be mentioned that the bag model curve strongly violates the inequality $h_1^u(x) < u(x)$, if one uses the experimental values for $u(x)$.

In order to establish an input for comparison with data of future experimental studies as well as for possible renormalization group analysis we suggest the following ansatz for $h_1^p(x)$ in the whole region of Bjorken variable, based on our calculation. We recall first that the behaviour of structure functions at $x \to 0$ is governed by the Regge trajectory in the t-channel of the corresponding forward scattering amplitude (see for details, [11]) so that, in particular,

$$h_1^p(x) \sim x^{-\alpha_{a_1}(0)} \tag{36}$$

where $\alpha_{a_1}(0)$ is the intercept of the axial a_1-meson trajectory. Assuming that this trajectory is linear and has the same slope as ρ, a_2 trajectories, namely $\alpha' \simeq 1GeV^{-2}$, and using the mass of the a_1 meson we easily get $\alpha_{a_1}(0) \simeq -0.3$. We then extrapolate $h_1^p(x)$ from the point $x = 0.3$, the minimal value at which our calculation is reliable, to the region $0 < x < 0.3$. using our result for $h_1^u(x = 0.3)$ and (35),(36). Note that Regge behaviour was also invoked in [2] for analysis of the moments of $h_1(x)$.

In the large x region where the structure function should decrease according to the general quark counting rules, we simply use the inequality $u(x) > h_1^u(x)$ established in [2] and replace $h_1^u(x)$ by the corresponding values of the $u(x)$ distribution (taking it, e.g. from [12])beginning from the point $x = 0.55$ where $h_1^u(x)$ starts to violate this inequality. (Within the uncertainties of $h_1^u(x)$ and $u(x)$ this point may be as large as $x \simeq 0.7$, the upper limit of the interval of reliability of our calculations). The resulting curve is shown in Fig.6.

Finally, let us compare our predictions for $h_1(x)$ with the nucleon spin structure function $g_1(x)$. Using recent data of SLAC E142 experiment [13] on the neutron spin structure function $g_1^n(x)$ measured at $Q^2 > 1GeV^2$ we conclude that at $x > 0.3$ this structure function values are consistent with zero. In terms of separate u and d quark spin distributions in the proton, the following relation should be valid starting from $x = 0.3$: $g_1^u(x) \simeq -4g_1^d(x)$ clearly indicating that the the proton spin distribution $g_1^p(x)$, as well as $h_1^p(x)$, is dominated at $x > 0.3$ by the u-quark contribution:

$$g_1^p(x) \simeq (4/9)g_1^u(x) \ . \tag{37}$$

Taking experimental results on $g_1^p(x)$ from EMC measurement [14] we get, e.g. $g_1^u(x) \simeq 0.5$ (0.2) at $x = 0.3$ ($x = 0.5$). The recent SMC measurement [15] gives the values of $g_1^u(x)$ at $x = 0.3$ and 0.5 about 20% lower, but agreeing with EMC within errors. Comparison with our calculation clearly indicates the validity of the inequality for u quark distributions in the proton:

$$h_1^u(x) > g_1^u(x) \tag{38}$$

at $x \geq 0.3$. The same conclusion can be obtained, if we compare $h_1(x)$ with the structure function $g_1(x)$ calculated in [8] in the framework of the method, similar to used here. Unfortunately, the $g_1(x)$ calculation is legitimate only in the narrow domain of x, $0.5 < x < 0.7$. In this domain the results of [8] are in good agreement with recent data of SMC [15]. In the interval $0.4 < x < 0.7$ the result of this experiment for the mean value of the proton spin structure function is $\overline{g_1^p(x)} = 0.08 \pm 0.02 \pm 0.01$ (see Table 1 in [15]). The prediction of ref. [8] for the same interval of x was $\overline{g_1^p(x)} = 0.06$ with an estimated error about 30% (some extrapolation of the $g_1^p(x)$ curve from $x = 0.5$ to $x = 0.4$ is done). As was shown in [8], the u-quark contribution to the spin structure function dominates in this region of x, in accordance with (37). Using the results of [8] we can compare theoretical predictions of $h_1^u(x)$ and $g_1^u(x)$ at the point $x = 0.5$ (but not at $x = 0.3$). We find from [8] $g_1^u(0.5) \simeq 0.14$ which agrees within theoretical and experimental errors with the value obtained above from EMC experiment and with inequality (38). The results of bag model calculations [2] also obey (38).

6 Conclusion

In this paper a new method of calculation of the proton chirality violating structure function $h_1^p(x)$ at intermediate x has been developed based on the QCD sum rule approach. The obtained estimates for this structure function are valid at $0.3 < x < 0.7$. Since we disregard the perturbative QCD correction, our results are valid at intermediate $Q^2 \approx 5 - 10GeV^2$, where deviations from scaling are inessential. In our approach $h_1^p(x)$ is proportional to the quark condensate. We have calculated only two terms in the OPE. Nevertheless, once the method is established, the accuracy may be improved in future taking into account higher

order terms of OPE. We estimate the current uncertainty of our calculation at the level of 30-50%. Therefore, having in mind that the similar calculation of the proton spin structure function $g_1(x)$ has reasonable agreement with experiment we consider our estimate of $h_1^p(x)$ as a reliable and useful guideline for experiments aimed at measuring this structure function.

We found that $h_1^p(x)$ is rather flat in the interval $0.3 < x < 0.7$ and not too much smaller than an upper limit $\mid h_1^u(x) \mid < u(x)$ derived in [2]. This circumstance gives a good chance for its experimental study. Another interesting feature of $h_1^p(x)$ is that the main contribution comes from u-quarks, the d-quark contribution being suppressed, besides of the d-quark charge, by a factor of order $\alpha_s/\pi \sim 0.1$ or is given by the contribution of higher dimension operators in OPE, which, probably, are also small. For this reason we expect that for the neutron at intermediate x

$$h_1^n(x) \approx (1/4) h_1^p(x)$$

The comparison of our $h_1^u(x)$ with the same function calculated by Jaffe and Ji in the bag model [2] shows that our $h_1^u(x)$ is by a factor of 2-3 smaller in the interval $0.3 < x < 0.5$ and matches with $h_1^u(x)$ at $x = 0.6$. The condition $\mid h_1(x) \mid > \mid g_1(x) \mid$ found in the bag model [2] is also fulfilled in our calculation.

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THE FIGURES

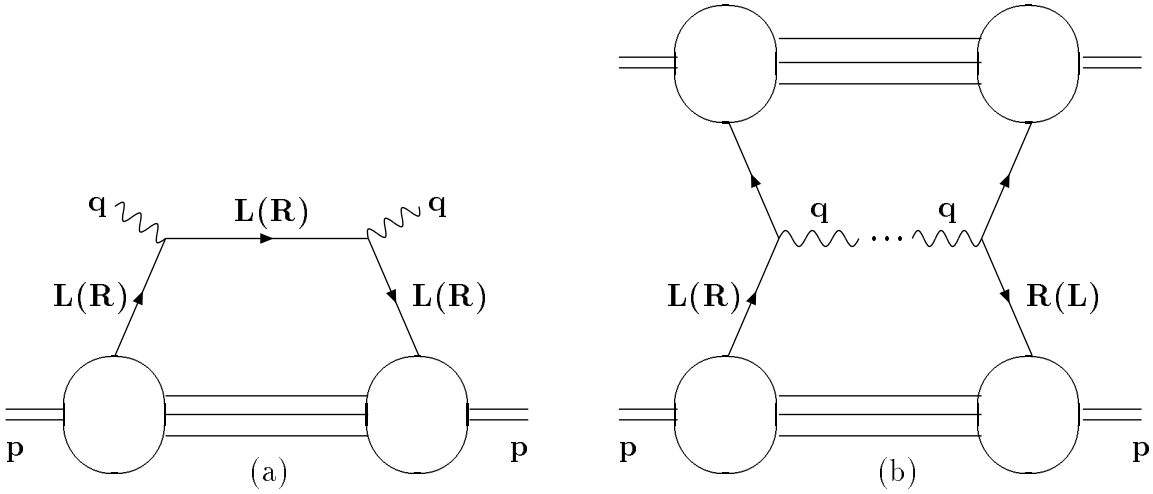

Figure 1: (a) Deep inelastic lepton-hadron scattering, the chirality of quarks is conserved. Solid lines are quarks, wavy lines are virtual photons, R(L) denote right (left) chirality of quarks; (b) Drell-Yan process with chirality of quarks flipped.

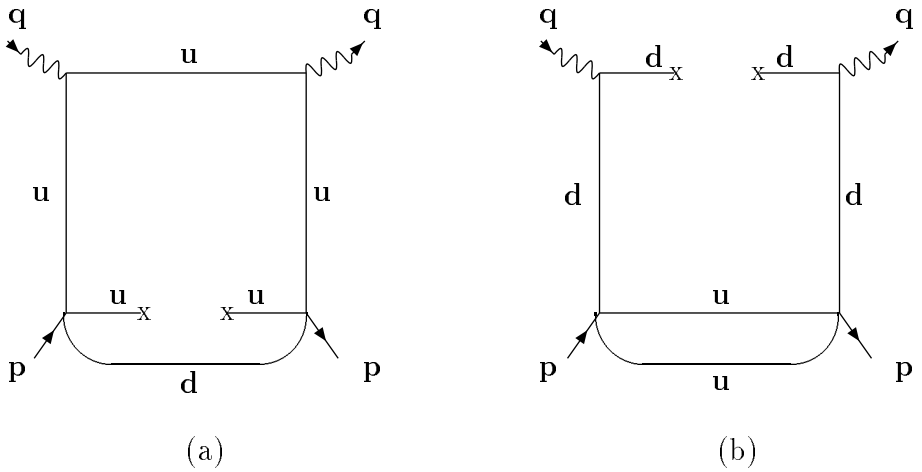

Figure 2: Diagrams corresponding to quark condensate contribution to the four-point correlator Π_μ defined in (12): a) leading quark condensate contribution from the scattering on u-quark ;b) nonleading contribution from the scattering on d-quark. Wavy lines denote axial or scalar currents, external momenta are shown by arrows, lines with a cross denote the quark vacuum fields.

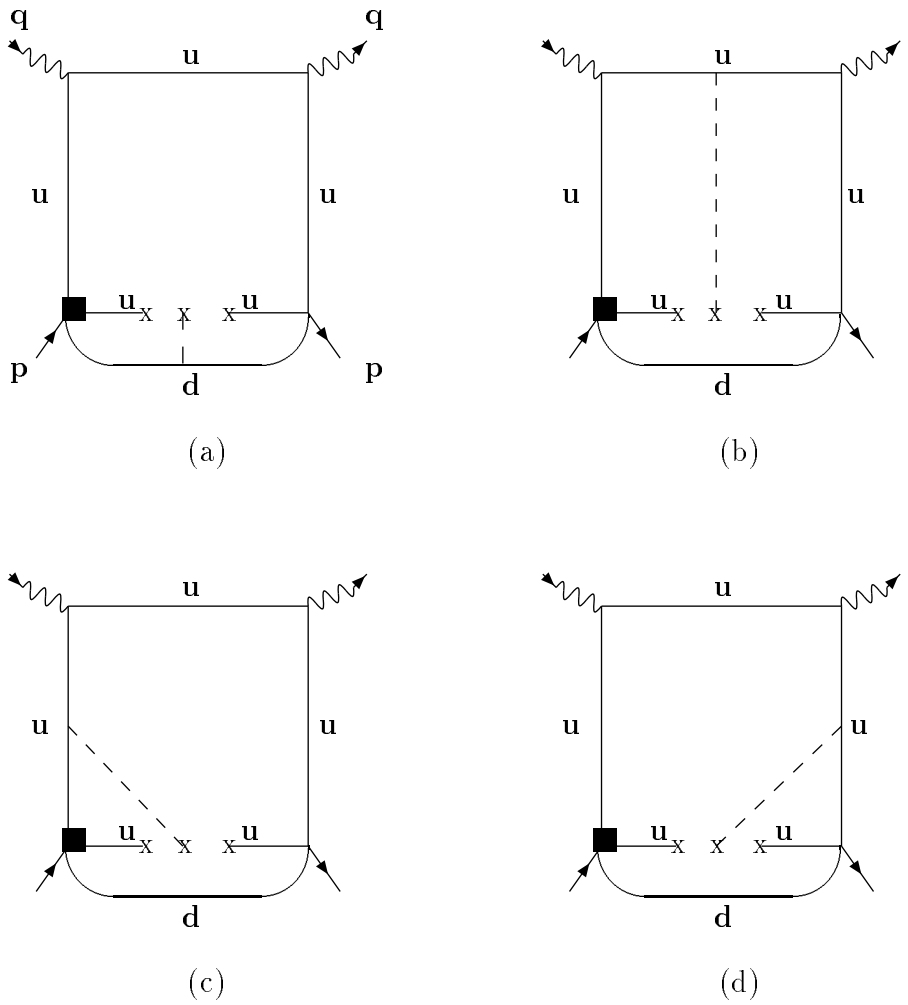

Figure 3: Diagrams corresponding to quark-gluon condensate contribution to the correlator (12). Dashed lines denote the vacuum gluon field. Black squares denote the fixed point of the gauge.

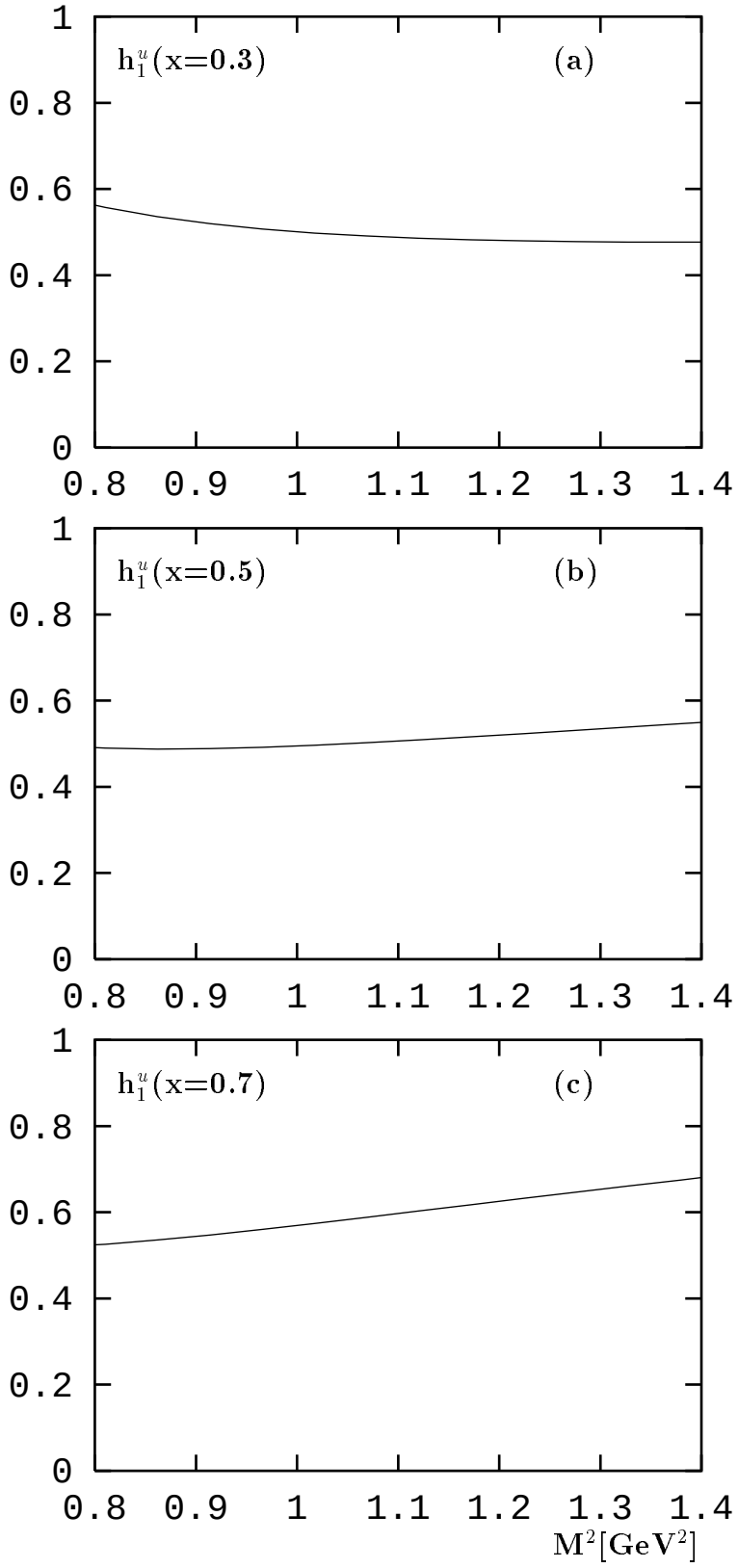

Figure 4: Dependence of u-quark contribution to $h_1(x)$ on Borel parameter M^2 at various values of x

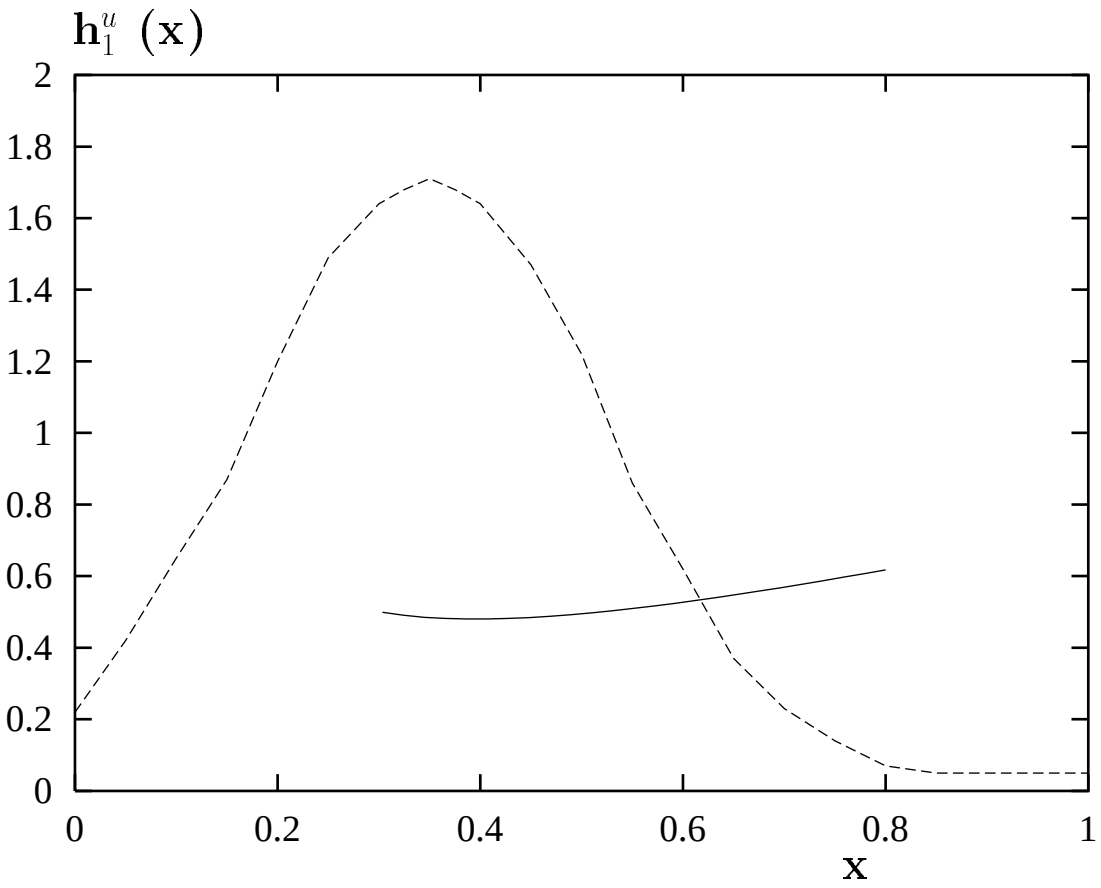

Figure 5: The u-quark contribution to the proton structure function $h_1(x)$ determined from QCD sum rule (34) at $M^2 = 1GeV^2$ (solid curve). The bag model prediction obtained by Jaffe and Ji [2] for the same structure function is shown by dashed line.

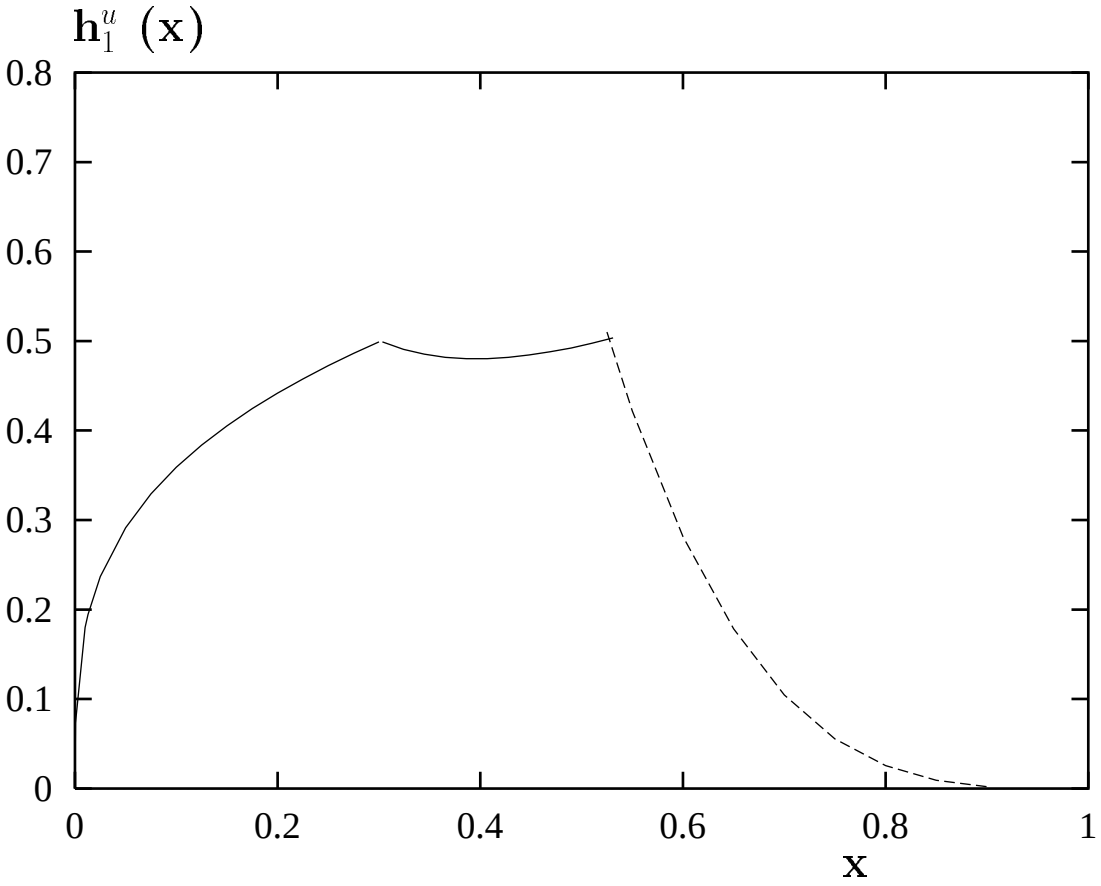

Figure 6: The u-quark contribution to the proton structure function $h_1(x)$ based on QCD calculation at intermediate values of x. In the region of low x an extrapolation from the point x=0.3 is done according to the expected Regge behaviour of this structure function. The dashed line shows the valence u-quark distribution taken from ref. [12] at $Q^2 = 5GeV^2$ which serves as an upper limit for $h_1^u(x)$ at large x.

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